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Curve-of-Growth Analysis by Using a Micro-Computer

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マイクロコンピューターを用いた成長曲線解析法

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要　旨

成長曲線法は，モデル大気法とともに，恒星の大気の分光分析の方法としてよく用いられる．この方法では，恒星の吸収線の励起ポテンシャルや測定された等価幅の値などをもとに描かれた経験成長曲線と最もよく一致する理論成長曲線を選び出すことにより，その恒星の大気の励起温度などの物理量を求めている．従来，この両曲線の一致は目測でなされていたので，得られた結果に個人差があり，また，誤差の客観的な見積りが困難だった．この欠点をなくすためにコンピューターを用いた解析もいくつか試みられてきている．

本研究では，過去になされた計算機による方法を改良し，マイコン用に変換したプログラムを用いた二つのタイプの成長曲線解析法が開発された．そして，Ba II 星の一つであるやぎ座ζ星に対して，過去にコンピューターを用いて相対成長曲線法でなされたのと同じデータを用いて，本研究で開発された二つの方法で解析がなされ，過去の結果と比べられた．その結果，一方の方法の優位性が示されたが，同時に，結果が理論成長曲線の選択に非常に大きく依存する場合のあることがわかった．

1. Introduction

A curve-of-growth analysis is one of the two methods which are mainly used for the analysis of stellar atmospheres. The other method is a model atmosphere analysis. Since detailed distributions of physical quantities in a stellar atmosphere are taken into account in the model atmosphere analysis, it is also called fine analysis. It is used when accurate observational data are available and the nature of the stellar atmosphere is known to a good approximation.

The curve-of-growth analysis is usually used when accurate observational data are not available or there is not enough knowledge about the nature of a stellar atmosphere. In this method, one-layer approximation is made, i.e. it is assumed that there exists a specific value for a physical quantity of the atmosphere such as pressure, temperature and density. A curve-of-growth is used in this method. The curve-of-growth is a graphical representation of the relation between the logarithm of the equivalent width of an absorption line log W and the logarithm of the number of absorbing atoms times the oscillator strength log Nf, where f is the oscillator strength and the equivalent width W

is the width of the rectangular profile for which the height is equal to the continuum level near the line and the area is that of the line. The equivalent width divided by wavelength W/λ is often used in stead of W, and some multiplicative factor is often added to Nf. We obtain by this method a representative value of the atmosphere for electron pressure, for gas pressure, for ionization temperature and for excitation temperature T_{ex} together with chemical composition. This method is also called coarse analysis. Furthermore, it is often called absolute curve-of-growth analysis or absolute coarse analysis in order to distinguish it from the method mentioned below.

In cases in which accurate values for oscillator strength are not known, the values for the abscissa log X_s of the curve-of-growth for the standard star for which the physical quantities and chemical composition of the atmosphere are already known are plotted instead of log Nf. In this case, the relative values to the standard star for the physical quantities and chemical composition are obtained instead of the absolute values. This method is called differential curve-of-growth analysis or differential coarse analysis.

The curve-of-growth analysis has conventionally been applied by eye measure. Thus, there has been a fear that the results obtained by this method depend on the subjectivity of an analyzer. Moreover, an objective estimate of error can not been made for this method. Recently, the curve-of-growth analyses by using a computer have been applied in order to overcome the above weak points. For example, Tech (1971)[1] has done a differential curve-of-growth analysis for Ba II star ζ Cap, using ε Vir (G8 III type star) as standard star. In this analysis, he determined the differential reciprocal excitation temperature $\Delta\theta_{ex}$ ($\theta_{ex}=5040/T_{ex}$) relative to the standard star by the minimum-sigma method, using a computer. Powell (1971)[2] has made computer programs for a differential curve-of-growth analysis of solar-type stars. However, these methods require a large amount of memory capacity and can be applied only to a large-sized computer.

Recently, micro-computers have been widely spread. In this paper, the curve-of-growth analysis by using a micro-computer has been developed. The micro-computer programs made in this paper have also been applied to ζ Cap using the same data as Tech (1971) and the results in this paper have been compared with those obtained by Tech (1971).

2. The Conventional Procedure of the Curve-of-Growth Analysis

The most important processes in the curve-of-growth analysis are the fitting the points of absorption lines in the curve-of-growth plane to a theoretical curve-of-growth and the determination of θ_{ex} or of $\Delta\theta_{ex}$. In the one-layer approximation made in the curve-of-growth analysis, a Boltzmann correction accounting for the differing populations of the lower energy levels of absorption lines appears as an additive term in the expression for the abscissa of the curve-of-growth. In absolute curves-of-growth, that is to say, the quantity plotted along the abscissa $\log X_{abs}$ is

$$\log X_{abs}=\log(gf\lambda)-\theta_{ex}\chi_1, \tag{1}$$

and in differential curve-of-growth the quantity along the abscissa $\log X_{rel}$ is

$$\log X_{rel}=\log X_s-\Delta\theta_{ex}\chi_1, \tag{2}$$

where g is the statistical weight of the lower energy level and χ_1 is the excitation potential of the lower energy level. Moreover, in the one-layer approximation, it is usually assumed that a single curve-of-growth can be applied to neutral atoms and to ionized atoms.

The above processes have been conventionally done in the following way. First, separate curves-of-growth are prepared for absorption lines of the atom at the same stage of ionization in each of several small excitation ranges. Then, each of these empirical curve-of-growth is fitted by eye measure to a theoretical curve-of-growth. In this fitting, it is assumed that the Boltzmann correction remains effectively constant for each empirical curve-of-growth, for the excitation range of the lines plotted for each curve is small. The horizontal shifts required to fit the separate curves to the theoretical curve are, according to the expression (1) or (2), linearly related to the mean excitation potential of the lines plotted for the separate curves. The gradient of this linear function is taken to be θ_{ex} or $\Delta\theta_{ex}$.

The above procedure has the following weak points: 1) It is necessary to have sufficient number of lines for each excitation range to enable comparison with theoretical curves-of-growth; 2) The excitation ranges are rather large ($\sim$1eV) quite often and this leads to a rather large horizontal spread ($\sim$1dex) due to temperature alone in the separate absolute curves-of-growth; 3) The derived θ_{ex} or $\Delta\theta_{ex}$ depends both on the way in which the available lines are separated into excitation ranges and on the way in which the fittings are made by eye measure.

3. The Procedure by Using a Computer Done to Date

Several trials have been done to overcome the above weak points by using a computer. Two typical examples are explained in the following.

3.1. The Minimum-Sigma Method by Tech

Tech (1971) has done a differential curve-of-growth analysis by using a computer. His procedure which he called the minimum-sigma method are made in the following way.

A preliminary value for $\Delta\theta_{ex}$ is chosen and the value log X_{rel} is calculated according to the expression (2) for each line of a given element at the same ionization stage. Then, a mean curve of cubic or quartic polynominal is calculated by the least-squares method to give the best representation of log X_{rel} as a function of log (W/λ), and the standard deviation σ of the points from this mean curve in a direction parallel to the log X_{rel} axis is calculated. By repeating the above calculation for several values of $\Delta\theta_{ex}$, a correlation between σ and $\Delta\theta_{ex}$ is obtained. A graph of this correlation is generally a smooth curve with a unique minimum. The adopted value of $\Delta\theta_{ex}$ is taken to be that value for which σ is least. Using this value of $\Delta\theta_{ex}$, the empirical curve-of-growth is reconstructed by plotting for each line log X_{rel} along the abscissa and log (W/λ) along the ordinate. Then, this empirical curve-of-growth is fitted to the theoretical curve-of-growth and the horizontal shift of this empirical curve onto the theoretical one gives the quantity which is related to the ratio of the number density of the element at the ionization stage concerned between the star being analyzed and the standard star.

The theoretical curve for ζ Cap adopted by Tech (1971) was that for pure absorption in a Milne-Eddington atmosphere calculated by Hunger (1956)[3] with damping parameter log $(2\alpha)=-2.5$ and with log $(c/2R_c v_D)=4.63$, where c is the speed of light and v_D is the Doppler velocity; Rc is the limiting central depth for strong lines. In the paper by Tech (1971)[1], he wrote, "The theoretical curve that offers the best fit to the majority of the empirical curves of growth for ζ Cap, and the one that has been adopted, is that for pure absorption in a Milne-Eddington atmosphere...", but he did not describe the details of the fitting, e.g. the criterion of the best fit.

As a test of this procedure, Tech (1971) has determined $\Delta\theta_{ex}$ value of ε Vir relative to the sun from neutral iron lines. His result of 0.19 is in good agreement with the value 0.18 derived by Cayrel and Cayrel (1963)[4] and 0.17 by Nishimura (1967)[5].

The strong points of this procedure, which is the reversal of the weak points of the conventional procedure, are as follows: 1) Each line is treated separately and separate weight can be applied to each line; 2) Correct excitation potentials rather than mean values of excitation ranges are taken into account; 3) It gives dispassionately reproducible results and objective estimates of error.

On the other hand, this procedure has the following weak points: 1) Great care must be exercised in assuring that no widely discordant lines are used; 2) Since lines on the flat or damping portions of the curve-of-growth will dominate the value of σ and mask the variation due to $\Delta\theta_{ex}$, such lines are generally excluded in the analysis, which brings about ambiguity to the results; 3) There is not a guarantee that the mean curve from which the σ values are calculated really represents the distribution of points adequately.

3.2. The Procedure by Powell

Powell (1971) has made computer programs for a differential curve-of-growth analysis, using the sun as a standard star. The procedure in these programs is based on the formulae by Pagel (1964)[6], that is, the abscissa in a curve-of-growth log X is normalized so that $\log X = \log (W/\lambda)$ for sufficiently weak lines, and for neutral lines, the quantity plotted along the abscissa in an empirical curve-of-growth is not the right side of the equation (2) but $\log X_s + \Delta\theta_{ex}\Delta\chi$, where $\Delta\chi$ is the difference between the ionization potential and the lower excitation potential.

In this procedure, the $\Delta\theta_{ex}$ value and the vertical and horizontal shifts to fit an empirical curve to a theoretical one are first determined, and then the shape of the theoretical curve which fits best to the empirical one, i.e. the damping parameter of the theoretical one is determined.

In the determinations of the $\Delta\theta_{ex}$ value and these vertical and horizontal shifts, only the lines which are on the linear portion or on the knee of the flat portion of the curve-of-growth are used, because the $\Delta\theta_{ex}$ value determined from these lines depends only slightly on the shape of the theoretical curve and is not affected very much by the vertical shift adopted in the fitting.

The determinations are done in the following iterative way. First, an initial value of $\Delta\theta_{ex}$ is estimated and the empirical curve-of-growth is constructed. Secondly, the empirical curve is fitted to the theoretical one by van der Held (1931)[7] with a damping parameter $\alpha = 0.05$. Van der Held curves-of-growth are those for pure absorption in a Schuster-Schwarzschild atmosphere and Cowley and Cowley (1964)[8] has found that an absolute curve-of-growth for the sun constructed by them fits best to the van der Held curve with $\alpha = 0.05$. Thirdly, the theoretical curve fitted to the empirical one is further shifted horizontally in order to normalize it so that it passes through the point $(-6.5, -6.5)$, and the value of log X corresponding to $\log (W/\lambda)$ for the star being analzed is read off for each line from this normalized curve. Lastly, a new value of $\Delta\theta_{ex}$ is found from a least-squares solution to the relation,

$$[X] = [A] + \Delta\theta_{ex}\Delta\chi, \tag{3}$$

where square brackets represent the logarithmic difference of the denoted quantity between the star being analyzed and the sun; A is the number ratio of a relevant element and to hydrogen uncorrected for ionization. The above process is repeated until a difference between successive estimates of $\Delta\theta_{ex}$ becomes less than 0.005.

Adopting the values of $\Delta\theta_{ex}$ and of the vertical and horizontal shifts thus determined, the final value of α is determined by obtaining the best fit of the empirical curve to a family of van der Held curves on the condition for a least-squares fit in a direction parallel to the $\log (W/\lambda)$ axis for all the points in the curve-of-growth. If this value of α is more than a factor of ten greater or less than 0.05, the above process of the determinations of the $\Delta\theta_{ex}$ value and the shifts is repeated using the van der Held curve with the new value of α.

In the above process of the determinations of the $\Delta\theta_{ex}$ value etc., the fitting of the van der Held curve to the empirical one is done on the assumption that the values for the abscissa are accurately known and the values for the ordinate have a Gaussian error distribution. Consequently, the fitting is done on the condition for a least-squares fit in a direction parallel to the log (W/λ) axis. This fitting is done in the following way which is also iterative. First, the initial value of the vertical shift Δy_i is taken to be zero and the initial value of the horizontal shift Δx_i is taken to be the mean value of maximum and minimum values of log $(W/\lambda)-\log X_s-\Delta\theta_{ex}\Delta\chi$. Secondly, values of R are calculated for two Δx values of $\Delta x_i+0.15$ and of $\Delta x_i-0.15$, where R is the derivative with respect to Δx of the sum of the squares of the deviation in the ordinate of the empirical curve from the van der Held curve which is shifted horizontally by Δx and shifted vertically by Δy. Thirdly, the Δx value corresponding to $R=0$ is estimated from the two R values and from the two Δx values by assuming that a linear relation between R and Δx exists. Lastly, using this Δx value instead of Δx_i, a new estimation of the Δx value such that $R=0$ is done. This iteration is continued until the difference between successive estimates of Δx is less than 0.0002. Using the final Δx value, a new value of Δy is estimated as a least-squares solution in a direction parallel to the ordinate. The whole process is repeated until the difference between successive estimates of Δy is less than 0.00002.

The strong points of this procedure are the same as described for the minimum-sigma method. The weak points of this procedure also are the same as the minimum-sigma method, except for the third point. There is, however, another weak point that there is a fear of divergence in the iterative process.

4. New Procedures by Using a Micro-Computer

Two new procedures of curve-of-growth analyses by using a micro-computer have been developed in this paper. In the new procedures, the merits of the procedures by Tech (1971) and by Powell (1971) have been made use of and developed for use with a micro-computer. A micro-computer PC-9801 (NEC) has been used throughout the analysis. The new procedures, which will be called the method of type 1 and the method of type 2, respectively, are done in the following ways.

4.1. The Method of Type 1

The method of type 1 consists of two processes. The first process is the fitting of theoretical curves-of-growth to an empirical curve. In this process, the shape of the theoretical curve which fits best to the empirical one is determined, along with the vertical shifts. The second process is the determinations of the $\Delta\theta_{ex}$ value and the horizontal shift, using the theoretical curve and adopting the vertical shift determined in the first process.

In the first process, the fitting is done on the assumption that the values for the abscissa are accurately known and the values for the ordinate have a Gaussian error distribution. Consequently, the fitting is done on the condition for a least-squares fit in a direction parallel to the ordinate. The theoretical curves fitted to the empirical one are those for pure absorption in a Milne-Eddington atmosphere calculated by Hunger (1956). The reasons why these curves are taken are as follows: 1) Hunger (1956) recommended to use these curves as well as those for coherent scattering in a Schuster-Schwarzschild atmosphere; 2) As quoted above, Tech (1971) has found that, among several types of theoretical curves, one of these curves fits best to the majority of the empirical curves for ζ Cap; 3) The author has found that, in a differential curve-of-growth analysis for ε Vir, the empirical curve of Fe I for ε Vir fits more to these curves than to those for coherent scattering in a Schuster-Schwarzschild atmosphere (Yoshioka (1979)[9)]).

The first process proceeds in the following way. First, a damping parameter of the theoretical curve fitted to the empirical one is assumed. Secondly, a range of the $\Delta\theta_{ex}$ values is setted. Thirdly, for each value of $\Delta\theta_{ex}$ in this range, the horizontal and vertical shifts are determined as a least-squares solution in a direction parallel to the ordinate, and the standard deviation for the solution. The stepping value of $\Delta\theta_{ex}$ is usually taken to be 0.01. The above process is repeated for other values of damping parameter, and the damping parameter and the vertical shift for which the standard deviation is minimum are adopted as the final values for these quantities.

In the second process, the determinations are done on the assumption that values for the ordinate are accurately known and the values for the abscissa have a Gaussian error distribution. Consequently, the values of $\Delta\theta_{ex}$ and of the horizontal shift are determined as least-squares solutions in a direction parallel to the abscissa.

The program for the first process written in BASIC is listed in Appendix 1, which program is named "COG1". In this program, the determination of the horizontal shift Δx and vertical shift Δy for given values of $\Delta\theta_{ex}$ and damping parameter is done as follows. First, two Δx values are setted, and the Δy values and the standard deviations σ for the two Δx values and for the mean value of these two Δx values are calculated. Secondly, the Δx value which give a minimum σ value Δx_e is estimated by assuming that σ is a quadratic function of Δx. Lastly, the Δy values and the σ values are calculated for the Δx values ranging from $\Delta x_e - 0.09$ to $\Delta x_e + 0.09$, where the stepping value of Δx is usually taken to be 0.01, and the Δx value and the corresponding Δy value which give a minimum σ value are selected.

The program for the second process also written in BASIC is listed in Appendix 2, which program is named "COG3". In this program, a gradient of the theoretical curve-of-growth for the ordinate of a line is taken into account as a weight for a least-squares solution. The program in which the gradient is not taken into account has also been made and it is named "COG2".

The above programs are based on the formulae by Pagel (1964).

4.2. The Method of Type 2

In the method of type 2, the value of $\Delta\theta_{ex}$ and the values of Δx and Δy are determined simultaneously with the value of damping parameter, using only the program "COG2" or "COG3".

These values are determined in the following way. First, the value of damping parameter is setted. Secondly, the values of $\Delta\theta_{ex}$ and of Δx are determined as least-squares solutions in the direction parallel to the abscissa for various values of Δy. Thirdly, the Δy value and the corresponding $\Delta\theta_{ex}$ and Δx values which give a minimum value of the standard deviation σ_{temp} of the $\Delta\theta$ value are selected. The above process is repeated for various values of damping parameter, and the value of damping parameter and the corresponding values of $\Delta\theta_{ex}$, Δx and Δy for which the σ_{temp} value is minimum are adopted as the final values for these quantities.

5. Results and Discussion

As a test of this new procedure, a differential curve-of-growth analysis for ζ Cap relative to ε Vir has been done, using the same data as Tech (1971). The $\Delta\theta_{ex}$ values determined by Tech (1971) from Fe I, Ni I, Ti I and Cr I lines are listed in Table 1. The numbers of lines used by him are not the total number available. It seems quite probable that he used weak lines with good quality, but regrettably, he did not describe the criteria for selecting lines. Then these criteria have been estimated and are described in Table 2. In this table, the general quality Q estimated by Tech (1971) represents a degree of reliability of the value of log (W/λ) on a scale of 0 (very poor) to 5 (excellent).

The results by the new procedures are given and are compared with the results by Tech (1971) in Tables 3, 4, 5 and 6. Tables 3 and 4 give the results by the method of type 1, where Table 4 gives the result by considering a gradient of the curve-of-growth also as a weight and Table 4 gives the one by neglecting the gradient. Tables 5 and 6 give the results by the method of type 2 by neglecting and by considering the gradient, respectively. In these tables, the results by using all the lines of the species are given in the columns

Table 1. Differential reciprocal excitation temperatures of ζ Cap relative to ε Vir determined by Tech.

Spectrum	Number of lines	$\Delta\theta_{ex}$
Fe I	189	−0.09
Ni I	24	−0.09
Ti I	34	−0.08
Cr I	21	−0.14

Table 2. Criteria for weak lines.

Spectrum	Number of lines		Upper limit of log (W/λ)	Upper limit of log X_s	Lower limit of Q
	All lines	Weak lines			
Fe I	234	189	no	−3.2	1
Ni I	44	24	−4.9	no	1
Ti I	41	34	−4.48	no	no
Cr I	31	21	−4.5	no	1

Table 3. Differential reciprocal excitation temperature of ζ Cap relative to ε Vir determined by the method of type 1. The theoretical curve-of-growth is determined from Fe I lines and has the damping parameter of log $(2\alpha)=-3.0$ and the vertical shift of log $(c/2R_c v_D)=4.71$. The gradient of the curve-of-growth is not taken into account as a weight.

Spectrum	All lines		Weak lines	
	Weight	$\Delta\theta_{ex}\pm$p. e.	Weight	$\Delta\theta_{ex}\pm$p. e.
Fe I	$Q+1$	−0.02±0.0101 (+0.07)	Q	−0.02±0.0107 (+0.07)
Ni I	$Q+1$	−0.02±0.0192 (+0.07)	Q	−0.07±0.0161 (+0.07)
Ti I	$Q+1$	−0.01±0.0316 (+0.07)	$Q+1$	−0.11±0.0221 (−0.03)
Cr I	$Q+1$	+0.04±0.0355 (+0.18)	Q	−0.12±0.0264 (+0.02)

Table 4. Differential reciprocal excitation temperature of ζ Cap relative to ε Vir determined by the method of type 1. The theoretical curve-of-growth is determined from Fe I lines and has the damping parameter of log $(2\alpha)=-3.0$ and the vertical shift of log $(c/2R_C v_D)=4.71$. The gradient of the curve-of-growth is also taken into account as a weight.

Spectrum	All lines		Weak lines	
	Weight	$\Delta\theta_{ex}\pm$p. e.	Weight	$\Delta\theta_{ex}\pm$p. e.
Fe I	$Q+1$	-0.03 ± 0.0085	Q	-0.03 ± 0.0092
		$(+0.06)$		$(+0.06)$
Ni I	$Q+1$	-0.04 ± 0.0176	Q	-0.07 ± 0.0168
		$(+0.05)$		$(+0.02)$
Ti I	$Q+1$	-0.05 ± 0.0237	$Q+1$	-0.09 ± 0.0211
		$(+0.03)$		(-0.01)
Cr I	$Q+1$	-0.01 ± 0.0294	Q	-0.09 ± 0.0277
		$(+0.13)$		$(+0.04)$

Table 5. Differential reciprocal excitation temperature of ζ Cap relative to ε Vir determined by the method of type 2. The gradient of the curve-of-growth is not taken into account as a weight.

Spectrum	All lines				Weak lines			
	Weight	Theoretical curve		$\Delta\theta_{ex}\pm$p. e.	Weight	Theoretical curve		$\Delta\theta_{ex}\pm$p. e.
		log 2α	log $(c/2R_c v_D)$			leg 2α	log $(c/2R_c v_D)$	
Fe I	$Q+1$	-2.6	4.72	-0.04 ± 0.00989	Q	-0.7	4.78	-0.04 ± 0.00887
				$(+0.05)$				$(+0.05)$
Ni I	$Q+1$	-2.8	4.57	-0.05 ± 0.01621	Q	-1.7	4.26	-0.08 ± 0.01519
				$(+0.04)$				$(+0.01)$
Ti I	$Q+1$	-1.2	4.71	-0.12 ± 0.02273	$Q+1$	-2.8	4.72	-0.11 ± 0.02203
				(-0.04)				(-0.03)
Cr I	$Q+1$	-2.2	4.61	-0.12 ± 0.02851	Q	-3.1	4.70	-0.12 ± 0.02639
				$(+0.02)$				$(+0.02)$

designated as all lines, and the results by using weak lines are given in the columns designated as weak lines, where "weak lines" means the lines which satisfy the criteria described in Table 2. The values given in parentheses in these tables are the differences between our results and those by Tech (1971).

Figure 1 shows the curve-of-growth of Fe I obtained by the method of type 1, using all lines and considering the gradient. Figure 2 shows the corresponding correlation of Fe I lines between the horizontal shift and the $\Delta\chi$ value.

Table 6. Differential reciprocal excitation temperature of ζ Cap relative to ε Vir determined by the method of type 2. The gradient of the curve-of-growth is also taken into account as a weight.

Spectrum	All lines				Weak lines			
	Weight	Teoretical curve		$\Delta\theta_{ex}\pm$p. e.	Weight	Theoretical curve		$\Delta\theta_{ex}\pm$p. e.
		log 2α	log $(c/2R_c v_D)$			log 2α	log $(c/2R_c v_D)$	
Fe I	$Q+1$	-2.7	4.71	-0.04 ± 0.00835 (+0.05)	Q	-2.5	4.63	-0.05 ± 0.00837 (+0.04)
Ni I	$Q+1$	-3.3	4.57	-0.06 ± 0.01521 (+0.03)	Q	-1.9	4.30	-0.08 ± 0.01540 (+0.01)
Ti I	$Q+1$	-1.3	4.71	-0.09 ± 0.02050 (-0.01)	$Q+1$	-2.8	4.72	-0.09 ± 0.02112 (-0.01)
Cr I	$Q+1$	-2.4	4.63	-0.09 ± 0.02499 (+0.05)	Q	-2.5	4.70	-0.10 ± 0.02770 (+0.04)

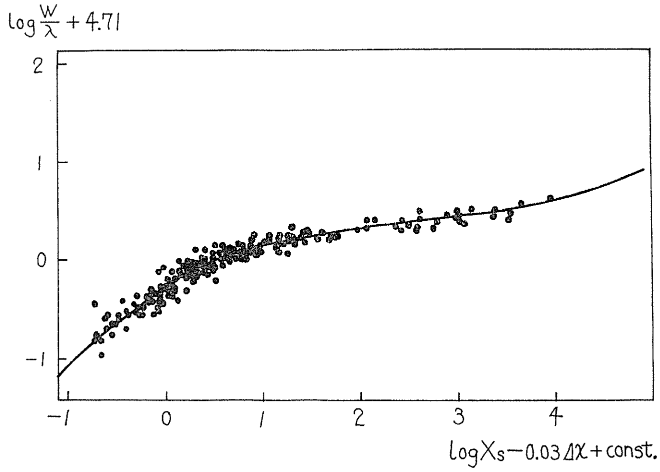

Fig. 1. Curve-of-growth of Fe I for ζ Cap. The solid line is the theoretical curve with log $(2\alpha)=-3.0$ and log $(c/2R_c v_D)=4.71$, which is selected by the method of type 1. The filled circles are plotted by adopting the $\Delta\theta_{ex}$ value as -0.03.

The following conclusions may be obtained from these tables and figures.

1) The method of type 2 by using all lines and by considering the gradient gives the results which best agree to the results by Tech (1971). This method also seems to desirable in priciple, because it is more consistent than the method of type 1 and the second weak point above described does not appear by using all lines. The bad example by using weak lines is shown in Figure 3.

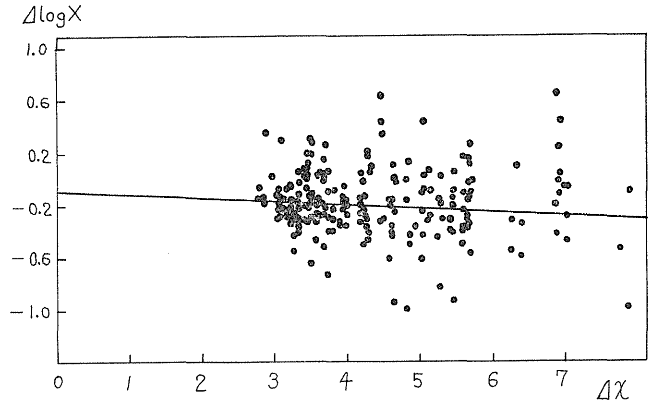

Fig. 2. Correlation between the horizontal shift and $\Delta\chi$ for all the Fe I lines of ζ Cap. The solid line shows the least-squares solution obtained by considering the gradient: $\Delta \log X=-0.03\,\Delta\chi-0.08$. The theoretical curve used is the one shown in Figure 1.

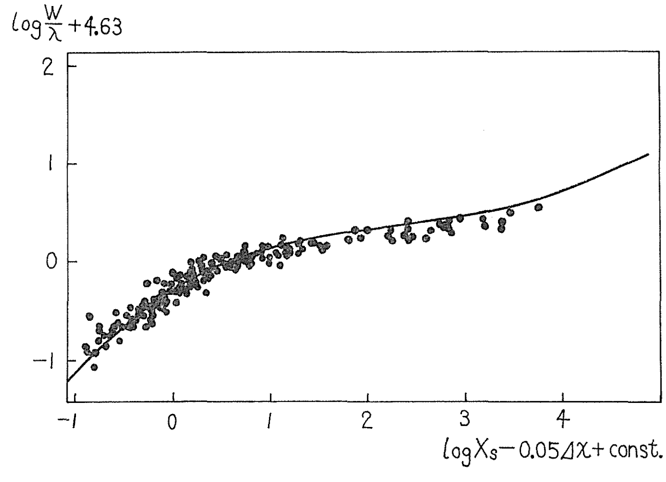

Fig. 3. Curve-of-growth of Fe I for ζ Cap. The solid line is the theoretical curve with log $(2\alpha)=-2.5$ and log $(c/2R_c v_D)=4.63$. Although this theoretical curve is selected by the method of type 2 from weak lines by considering the gradient, all lines are plotted, where the Δ_{ex} value are adopted as -0.05.

2) There is an unexpectedly strong dependence of the $\Delta\theta_{ex}$ value on the damping parameter and on the vertical shift. The strong dependence occurs especially for a group of lines where the $\Delta\chi$ value correlates strongly with the location of a curve-of-growth, as in the case of Cr I lines. The extreme cases for Cr I are shown in Figures 4 and 5. In these cases, even a sign of the

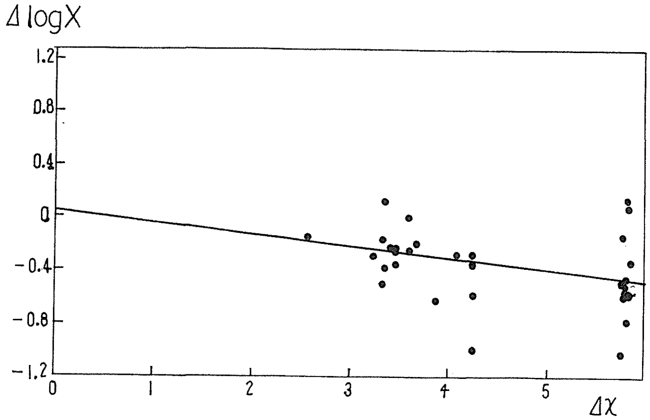

Fig. 4. Correlation between the horizontal shift and $\Delta\chi_{ex}$ for all the Cr I lines of ζ Cap. The solid line shows the least-squares solution obtained by neglecting the gradient: $\Delta \log X = -0.09\, \Delta\chi + 0.04$. The theoretical curve used is the one with $\log(2\alpha) = -2.5$ and $\log (c/2\, R_c v_D) = 4.63$.

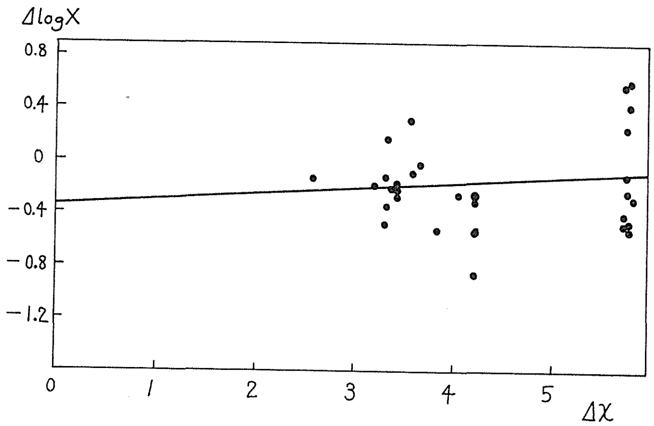

Fig. 5. Correlation between the horizontal shift and $\Delta\chi_{ex}$ for all the Cr I lines of ζ Cap. The solid line shows the least-squares solution obtained by neglecting the gradient: $\Delta \log X = 0.04\, \Delta\chi - 0.34$. The theoretical curve used is the one with $\log (2\alpha) = -3.0$ and $\log (c/2R_c v_D) = 4.71$.

$\Delta\theta_{ex}$ value is reversed, depending on the adopted values of damping parameter and of vertical shift.

3) Some $\Delta\theta_{ex}$ values obtained by the new procedures differ markedly from the $\Delta\theta_{ex}$ values by Tech (1971), as is shown in Table 7. The cause for

Table 7. Differential reciprocal excitation temperature of ζ Cap relative to ε Vir determined by using the same theoretical curve-of-growth as Tech, i.e., the curve with log $(2\alpha)=-2.5$ and with log $(c/2R_c v_D)=4.63$. The gradient of the curve-of-growth is also taken into account.

Spectrum	All lines		Weak lines	
	Weight	$\Delta\theta_{ex}\pm$p. e.	Weight	$\Delta\theta_{ex}\pm$p. e.
Fe I	$Q+1$	-0.09 ± 0.0096 (0.00)	Q	-0.05 ± 0.0084 (+0.04)
Ni I	$Q+1$	-0.05 ± 0.0156 (+0.04)	Q	-0.07 ± 0.0163 (+0.02)
Ti I	$Q+1$	-0.10 ± 0.0211 (−0.02)	$Q+1$	-0.11 ± 0.0234 (−0.03)
Cr I	$Q+1$	-0.09 ± 0.0250 (+0.05)	Q	-0.12 ± 0.0289 (+0.02)

the disagreement seems to be the differences in the used lines, which indicates the strong dependence of the results on the lines used.

On the basis of the above results, the improvements of the new procedures are being made by the author.

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Appendix 1. List of the program "COG1"

```
10 REM Determination of the Curve of Growth
20 REM The Selected Curve Minimizes the Value for the Sum
30 REM of the Squares of Differences along the Ordinates
40 DEFINT I-K:WIDTH 80,25
50 DIM LAMDA(500),POTEN(500),LGW(500),LGX(500),DKAI(500)
60 DIM W(500),LGXX(500),LGXY(500),LGWX(500),TATE(12),V(21),DLGX(21)
70 DIM ER(21),DDLGX(20),VV(20),TTHETA(20),PX(30),PY(30),ALFA(8)
80 DIM A(11,2),TTATE(12,8)
90 K0=0:AA=-1000:PAI=3.14159
100 INPUT "If neutral push N or ion push I";A$
110 OPEN "2:DATA2" FOR INPUT AS #1
120 IF EOF(1) THEN CLOSE #1:GOTO 170
130 INPUT #1,LAMDA(K0),IND,POTEN(K0),LGW(K0),LGX(K0),W(K0)
140 W(K0)=W(K0)+1
150 IF LGX(K0)>AA THEN AA=LGX(K0):K1=K0
160 K0=K0+1:GOTO 120
170 K0=K0-1
180 OPEN "2:DATA1" FOR INPUT AS #1
190 I=0
200 IF EOF(1) THEN CLOSE #1:GOTO 240
210 INPUT #1,ALFA(I)
220 FOR J=0 TO 12:INPUT #1,TTATE(J,I):NEXT J
230 I=I+1:GOTO 200
240 WEIGHT=0:AB=0
250 FOR I=0 TO K0
260 IF A$="I" GOTO 280
270 DKAI(I)=7.87-POTEN(I):AB=AB+W(I)*DKAI(I):WEIGHT=WEIGHT+W(I):GOTO 290
280 DKAI(I)=-POTEN(I):AB=AB+W(I)*DKAI(I):WEIGHT=WEIGHT+W(I)
290 NEXT I
300 PR=SQR(K0*WEIGHT)/.67449
310 INPUT "log(2α)";AALFA
320 FOR I=0 TO 8
330 IF AALFA>ALFA(I) GOTO 350
340 NEXT I
350 S1=2*(AALFA-ALFA(I)):S2=S1-.5:S3=S1*(S1-1)
360 FOR J=0 TO 12
370 TATE(J)=(TTATE(J,I)+TTATE(J,I-1))/2
380 TATE(J)=TATE(J)+(TTATE(J,I-1)-TTATE(J,I))*S2
390 TATE(J)=TATE(J)+(TTATE(J,I-2)+TTATE(J,I+1)-TTATE(J,I-1)-TTATE(J,I))*S3/4
400 NEXT J
410 DX=AB/WEIGHT
420 LPRINT "log(2α)=";AALFA
430 LPRINT :LPRINT "  Δθ  log(c/2RV)     ΔlogA  prob.err   No.of J":LPRINT
440 INPUT "min(Δθ),max(Δθ),δ(Δθ)";TMIN,TMAX,DT
450 PRINT "5-max(logX)=";5-LGX(K1),"log(W/λ)=";LGW(K1),"mean Δχ=";DX
460 PRINT "max(ΔlogA)=";5-LGX(K1)-TMIN*DKAI(K1)
470 INPUT "min(ΔlogA),max(ΔlogA)";XMIN,XMAX
480 GOSUB *HENKANY
490 PRINT :PRINT "  Δθ  log(c/2RV)  ΔlogA  prob.err     No.of J":PRINT
500 THETA=TMIN
510 JJ=0
520 WHILE THETA<TMAX+.1*DT
530 JJ=JJ+1
540 FOR I=0 TO K0:LGXX(I)=LGX(I)+THETA*DKAI(I):NEXT I:GOTO 570
550 PRINT "Δθ=";THETA,"min(ΔlogA)=";XMIN,"max(ΔlogA)=";XMAX
560 INPUT "min(ΔlogA),max(ΔlogA)";XMIN,XMAX
570 GOSUB *KEISAN
580 GOSUB *SENTAK
590 PRINT USING "###.##";THETA;:PRINT SPC(2);:PRINT USING "##.##";V(J);:PRINT SPC(6);
600 PRINT USING "###.##";DLGX(J);:PRINT SPC(3);:PRINT USING "#.#####^^^^";ER(J)/PR;:PRINT SPC(2);J;
610 LPRINT USING "###.##";THETA;:LPRINT SPC(2);:LPRINT USING "##.##";V(J);:LPRINT SPC(6);
620 LPRINT USING "###.##";DLGX(J);:LPRINT SPC(3);:LPRINT USING "#.#####^^^^";ER(J)/PR;:LPRINT SPC(2);J;
630 IF J<4 OR J=21 THEN PRINT "outer point          XXMEAN=";XXMEAN:PRINT ELSE PRINT
640 IF J<4 OR J=21 THEN LPRINT "outer point          XXMEAN=";XXMEAN:LPRINT :GOTO 550 ELSE LPRINT
650 DDLGX(JJ)=DLGX(J):VV(JJ)=V(J):TTHETA(JJ)=THETA
660 THETA=THETA+DT
670 XMIN=DLGX(J)-.4-DX*DT:XMAX=DLGX(J)+.4-DX*DT
680 WEND
690 BEEP:BEEP:BEEP
700 INPUT "If you want the graph of the Curve-of-Growth, Push Y";ANS$
710 IF ANS$<>"Y" GOTO 800
720 INPUT "Δθ";THETA
730 FOR I=0 TO JJ
740 IF THETA>TTHETA(I)-.1*DT AND THETA<TTHETA(I)+.1*DT GOTO 760
750 NEXT I
760 GOSUB *GRAPH
770 ANS$=INKEY$
780 IF ANS$="" GOTO 770
790 CLS 3:CONSOLE 0,25,1,0
800 INPUT "If you repeat with other log(2a) or Δθ, Push Y";ANS$
810 IF ANS$="Y" THEN PRINT :PRINT :LPRINT :LPRINT :GOTO 310
820 END
830 *HENKANY
840 FOR I=1 TO 10
850 A(I,0)=(TATE(I+2)+TATE(I-1)-TATE(I+1)-TATE(I))/4
860 A(I,1)=(5*TATE(I+1)-3*TATE(I)-TATE(I+2)-TATE(I-1))/4
870 A(I,2)=TATE(I)
880 NEXT I
```

```
890 A(0,0)=(TATE(2)+TATE(0)-2*TATE(1))/2
900 A(0,1)=(4*TATE(1)-3*TATE(0)-TATE(2))/2
910 A(0,2)=TATE(0)
920 A(11,0)=(TATE(12)+TATE(10)-2*TATE(11))/2
930 A(11,1)=(TATE(12)-TATE(10))/2
940 A(11,2)=TATE(11)
950 RETURN
960 *KEISAN
970 XMEAN=(XMIN+XMAX)/2
980 DLGX(0)=XMAX:DLGX(1)=XMIN:DLGX(2)=XMEAN
990 FOR I=0 TO 2
1000 FOR J=0 TO K0:LGXY(J)=LGXX(J)+DLGX(I):NEXT J
1010 B=0:C=0
1020 FOR J=0 TO K0
1030 IF LGXY(J)<-1 GOTO 1150
1040 Z=-.5
1050 FOR K=0 TO 11
1060 IF LGXY(J)<=Z GOTO 1120
1070 Z=Z+.5
1080 NEXT K
1090 IF AALFA>=-2.001 THEN AC=EXP((LGXY(J)+AALFA)*LOG(10))/2:P=PAI*SQR(AC/SQR(PAI))/2:GOTO 1160
1100 PRINT "There is a line whose value for logX+Δθ+Δχ·ΔlogA is greater than 5"
1110 PRINT "logX+Δθ・Δχ+ΔlogA=";LGXY(J):GOTO 550
1120 P=A(K,0):Q=2*(LGXY(J)+.5-Z)
1130 FOR II=1 TO 2:P=P*Q+A(K,II):NEXT II
1140 GOTO 1160
1150 P=(LGXY(J)+10)*(TATE(0)+10.052)/9-10.052
1160 LGWX(J)=P
1170 B=B+W(J)*(P-LGW(J))
1180 NEXT J
1190 V(I)=B/WEIGHT
1200 FOR J=0 TO K0:C=C+W(J)*(V(I)+LGW(J)-LGWX(J))^2:NEXT J
1210 ER(I)=SQR(C)
1220 NEXT I
1230 XXMEAN=XMEAN+.25*(XMAX-XMIN)*(ER(1)-ER(0))/(ER(0)+ER(1)-2*ER(2))
1240 XXMIN=CINT(100*XXMEAN)/100-.09
1250 FOR I=0 TO 18
1260 B=0:C=0
1270 FOR J=0 TO K0:LGXY(J)=LGXX(J)+XXMIN:NEXT J
1280 FOR J=0 TO K0
1290 IF LGXY(J)<-1 GOTO 1430
1300 Z=-.5
1310 FOR K=0 TO 11
1320 IF LGXY(J)<=Z GOTO 1400
1330 Z=Z+.5
1340 NEXT K
1350 IF AALFA>=-2.001 THEN AC=EXP((LGXY(J)+AALFA)*LOG(10))/2:P=PAI*SQR(AC/SQR(PAI))/2:GOTO 1440
1360 XXMIN=XXMIN-.01*I
1370 PRINT "There is a line whose value for logX+Δθ・Δχ+ΔlogA is greater than 5"
1380 PRINT "logX+Δθ・Δχ+ΔlogA=";LGXY(J),"No.of I";I,"XXMIN=";XXMIN
1390 INPUT "min(ΔlogA)";XXMIN:I=0:GOTO 1250
1400 P=A(K,0):Q=2*(LGXY(J)+.5-Z)
1410 FOR II=1 TO 2:P=P*Q+A(K,II):NEXT II
1420 GOTO 1440
1430 P=(LGXY(J)+10)*(TATE(0)+10.052)/9-10.052
1440 LGWX(J)=P
1450 B=B+W(J)*(P-LGW(J))
1460 NEXT J
1470 V(I+3)=B/WEIGHT
1480 FOR J=0 TO K0:C=C+W(J)*(V(I+3)+LGW(J)-LGWX(J))^2:NEXT J
1490 ER(I+3)=SQR(C):DLGX(I+3)=XXMIN
1500 XXMIN=XXMIN+.01
1510 NEXT I
1520 RETURN
1530 *SENTAK
1540 IF XMIN>XXMIN-.191 AND XMIN<XXMIN-.009 THEN ER(1)=100000!
1550 IF XMAX>XXMIN-.191 AND XMAX<XXMIN-.009 THEN ER(0)=100000!
1560 IF (XMIN+XMAX)/2>XXMIN-.191 AND (XMIN+XMAX)/2<XXMIN-.009 THEN ER(2)=100000!
1570 ERMIN=100000!
1580 FOR I=0 TO 21
1590 IF ER(I)>ERMIN GOTO 1610
1600 J=I:ERMIN=ER(I)
1610 NEXT I
1620 RETURN
1630 *GRAPH
1640 SCREEN 2,0:COLOR 0
1650 CONSOLE 0,25,0,0:CLS 3
1660 FOR J=0 TO K0:LGXY(J)=LGX(J)+THETA*DKAI(J)+DDLGX(I):LGWX(J)=LGW(J)+VV(I):NEXT J
1670 LINE(20,380)-(630,380)
1680 LINE(20,20)-(20,380)
1690 FOR J=0 TO 3:LINE(17,40+J*100)-(20,40+J*100):NEXT J
1700 FOR J=0 TO 6:LINE(20+J*100,380)-(20+J*100,383):NEXT J
1710 LOCATE 58,23,0:PRINT "logC"
1720 LOCATE 2,24,0:PRINT -1;
1730 FOR J=1 TO 6:PRINT TAB(2+INT(J*12.5));:PRINT J-1;:NEXT J
1740 LOCATE 4,0,0:PRINT "Log(W/2RΔλ)      log2α=";AALFA;SPC(6);"log(c/2RV)=";VV(I)
1750 FOR J=0 TO 3:LOCATE 0,3+INT(6.25*J),1:PRINT 2-J:NEXT J
1760 FOR J=0 TO 30:PX(J)=-1+J*.2:NEXT J
1770 FOR J=0 TO 30
1780 IF PX(J)<-1 GOTO 1870
```

```
1790 Z=-.5
1800 FOR K=0 TO 11
1810 IF PX(J)<=Z GOTO 1840
1820 Z=Z+.5
1830 NEXT K
1840 P=A(K,0):Q=2*(PX(J)+.5-Z)
1850 FOR II=1 TO 2:P=P*Q+A(K,II):NEXT II
1860 GOTO 1880
1870 P=(PX(J)+10)*(TATE(0)+10.052)/9-10.052
1880 PY(J)=P
1890 NEXT J
1900 IPX1=CINT(100*PX(0))+120:IPY1=240-CINT(100*PY(0))
1910 FOR J=0 TO 29
1920 IPX2=CINT(100*PX(J+1))+120:IPY2=240-CINT(100*PY(J+1))
1930 LINE(IPX1,IPY1)-(IPX2,IPY2)
1940 IPX1=IPX2:IPY1=IPY2
1950 NEXT J
1960 FOR J=0 TO KO
1970 IPX=CINT(100*LGXY(J)):IPY=CINT(100*LGWX(J))
1980 IF IPX>500 OR IPY>220 GOTO 2020
1990 IF IPX<-100 OR IPY<-140 GOTO 2020
2000 IPX=IPX+120:IPY=240-IPY
2010 CIRCLE(IPX,IPY),3:PAINT(IPX,IPY)
2020 NEXT J
2030 RETURN
```

Appendix 2. List of the Program "COG3"

```
10 REM Determination of Δθ and <ΔlogA> for Single Species
20 REM The Selected Curve Minimizes the Value for the Sum
30 REM of the Squares of Differences along the Abscissa
40 REM The Gradient of the Curve Is Taken into Account for Weight
50 DEFINT I-K:WIDTH 80,25
60 DIM LAMDA(500),POTEN(500),LGW(500),LGX(500),DKAI(500)
70 DIM W(500),IW(500),DX(500),DDX(500),LGXA(500)
80 DIM ALFA(8),Y(11),TATE(12),TTATE(12,8),A(11,2)
90 INPUT "If neutral push N or ion push I";A$
100 INPUT "log(2α)";AALFA
110 OPEN "2:DATA1" FOR INPUT AS #1
120 I=0
130 IF EOF(1) THEN CLOSE #1:GOTO 170
140 INPUT #1,ALFA(I)
150 FOR J=0 TO 12:INPUT #1,TTATE(J,I):NEXT J
160 I=I+1:GOTO 130
170 FOR I=0 TO 8
180 IF AALFA>ALFA(I) GOTO 200
190 NEXT I
200 S1=2*(AALFA-ALFA(I)):S2=S1-.5:S3=S1*(S1-1)
210 FOR J=0 TO 12
220 TATE(J)=(TTATE(J,I)+TTATE(J,I-1))/2
230 TATE(J)=TATE(J)+(TTATE(J,I-1)-TTATE(J,I))*S2
240 TATE(J)=TATE(J)+(TTATE(J,I-2)+TTATE(J,I+1)-TTATE(J,I-1)-TTATE(J,I))*S3/4
250 NEXT J
260 GOSUB *HENKAX
270 INPUT "log(c/2RV)";V
280 I=0:JJ=0:DKAI1=0:DKAI2=0:IWW=0
290 OPEN "2:DATA2" FOR INPUT AS #1
300 IF EOF(1) THEN CLOSE #1:GOTO 390
310 INPUT #1,LAMDA(I),IND,POTEN(I),LGW(I),LGX(I),IW(I)
320 IW(I)=IW(I)+1
330 IF A$="I" GOTO 350
340 DKAI(I)=7.87-POTEN(I):GOTO 360
350 DKAI(I)=-POTEN(I)
360 LGW(I)=LGW(I)+V
370 IWW=IWW+IW(I)
380 I=I+1:JJ=JJ+1:GOTO 300
390 I=I-1:PX=0:PXY=0:WW=0
400 FOR J=0 TO I
410 YY=LGW(J):GOSUB *KEISAN:GOSUB *KEISAN1
420 DX(J)=X-V-.052-LGX(J):W(J)=IW(J)/DC:WW=WW+W(J)
430 NEXT J
440 WR=IWW/WW:WW=0
450 FOR J=0 TO I:W(J)=W(J)*WR:WW=WW+W(J):NEXT J
460 FOR J=0 TO I
470 DKAI1=DKAI1+W(J)*DKAI(J):DKAI2=DKAI2+W(J)*DKAI(J)^2
480 PX=PX+W(J)*DX(J):PXY=PXY+W(J)*DX(J)*DKAI(J)
490 NEXT J
500 D=DKAI2*WW-DKAI1^2
510 THETA=(PXY*WW-DKAI1*PX)/D:AA=(DKAI2*PX-DKAI1*PXY)/D
520 PP=0
530 FOR J=0 TO I
540 DDX(J)=DX(J)-THETA*DKAI(J)-AA:PP=PP+W(J)*DDX(J)^2
550 NEXT J
560 PETHETA=.67449*SQR(WW*PP/(JJ-2)/D):PEAA=.67449*SQR(DKAI2*PP/(JJ-2)/D)
570 BEEP:BEEP:BEEP
580 INPUT "Critical Difference";CD
590 CLS 3
600 LPRINT "log (2α) = ";AALFA,"log (c/2RV) = ";V,"Number of Lines = ";JJ:LPRINT
610 PRINT "Δθ  =";THETA,"Probable Error=";PETHETA
620 LPRINT "Δθ  =";THETA,"Probable Error=";PETHETA
630 PRINT " [A] = ";AA,"Probable Error=";PEAA:PRINT
640 LPRINT " [A] = ";AA,"Probable Error=";PEAA:PRINT
650 LPRINT :LPRINT "Critical Difference = ";CD:LPRINT
660 PRINT "Wavelength";SPC(5);"Log(W/λ)     LogX";SPC(11);"Difference":PRINT
670 LPRINT "Wavelength";SPC(5);"Log(W/λ)     LogX";SPC(11);"Difference":LPRINT
680 FOR J=0 TO I
690 IF ABS(DDX(J))<CD GOTO 720
700 PRINT LAMDA(J);SPC(7);LGW(J)-V;SPC(7);LGX(J);SPC(7);DDX(J)
710 LPRINT LAMDA(J);SPC(7);LGW(J)-V;SPC(7);LGX(J);SPC(7);DDX(J)
720 NEXT J
730 ANS$=INKEY$
740 IF ANS$="" GOTO 730
750 CLS 3
760 INPUT "Graphic (Relation [X] vs Δχ) Y or N";ANS$
770 IF ANS$="N" GOTO 810
780 GOSUB *GRAPH1
790 ANS$=INKEY$
800 IF ANS$="" GOTO 790
810 CLS 3:CONSOLE 0,25,1,0:LOCATE 0,0,1
820 INPUT "Graphic(Curve of Growth) Y or N";ANS$
830 IF ANS$="N" GOTO 880
840 GOSUB *GRAPH2
850 ANS$=INKEY$
860 IF ANS$="" GOTO 850
870 CLS 3:CONSOLE 0,25,1,0:LOCATE 0,0,1
880 END
```

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890 *HENKAX
900 REM LogC=A(I,0)*Y^2+A(I,1)*Y+A(I,2)         Y=log(W/(2·R·Δλ))
910 DIM E(10),F(10),G(10)
920 Z=-1
930 D=(TATE(0)-TATE(1))*(TATE(1)-TATE(2))*(TATE(2)-TATE(0))
940 E(0)=(TATE(1)-(TATE(0)+TATE(2))/2)/D
950 F(0)=((TATE(0)^2+TATE(2)^2)/2-TATE(1)^2)/D
960 G(0)=Z-(TATE(0)*(2*TATE(1)*(TATE(0)-TATE(1))+TATE(2)*(TATE(2)-TATE(0))))/(2*D)
970 A(0,0)=E(0):A(0,1)=F(0):A(0,2)=G(0)
980 FOR I=1 TO 10
990 Z=Z+.5
1000 D=(TATE(I)-TATE(I+1))*(TATE(I+1)-TATE(I+2))*(TATE(I+2)-TATE(I))
1010 E(I)=(TATE(I+1)-(TATE(I)+TATE(I+2))/2)/D
1020 F(I)=((TATE(I)^2+TATE(I+2)^2)/2-TATE(I+1)^2)/D
1030 G(I)=Z-(TATE(I)*(2*TATE(I+1)*(TATE(I)-TATE(I+1))+TATE(I+2)*(TATE(I+2)-TATE(I))))/(2*D)
1040 A(I,0)=(E(I-1)+E(I))/2:A(I,1)=(F(I-1)+F(I))/2:A(I,2)=(G(I-1)+G(I))/2
1050 NEXT I
1060 FOR I=0 TO 11:Y(I)=TATE(I+1):NEXT I
1070 A(11,0)=E(10):A(11,1)=F(10):A(11,2)=G(10)
1080 RETURN
1090 *KEISAN
1100 REM LogC is calculated from Log(W/(2·R·Δλ))
1110 IF YY=TATE(0) GOTO 1190
1120 FOR K=0 TO 11
1130 IF YY<=Y(K) GOTO 1160
1140 NEXT K
1150 X=-.1437+2*YY-AALFA+LOG(1+SQR(1+2.4674*(10^AALFA/10^YY)^2))/LOG(10):GOTO 1200
1160 X=A(K,0)
1170 FOR II=1 TO 2:X=X*YY+A(K,II):NEXT II
1180 RETURN
1190 X=9*(YY+10.052)/(TATE(0)+10.052)-10
1200 RETURN
1210 *GRAPH1
1220 CLS 3:SCREEN 2,0:COLOR 0:CONSOLE 0,25,0,0
1230 DKAIMAX=0:DXMIN=100:DXMAX=-100
1240 FOR J=0 TO I
1250 IF IW(J)=0 GOTO 1290
1260 IF ABS(DKAI(J))>DKAIMAX THEN DKAIMAX=ABS(DKAI(J))
1270 IF DX(J)<DXMIN THEN DXMIN=DX(J)
1280 IF DX(J)>DXMAX THEN DXMAX=DX(J)
1290 NEXT J
1300 PRINT "Max(ΔlogX)=";DXMAX,"Min(ΔlogX)=";DXMIN
1310 INPUT "Max.Graduation, Min.Graduation, Interval of Graduation";MAXGR,MINGR,DGR
1320 MSO=320/(MAXGR-MINGR):MSA=590/DKAIMAX:CLS 3
1330 LINE(40,345)-(635,345):LINE(40,20)-(40,345)
1340 IGO=INT((MAXGR-MINGR)/DGR+.1)
1350 FOR J=0 TO IGO
1360 LINE(37,25+CINT(MSO*J*DGR))-(40,25+CINT(MSO*J*DGR))
1370 NEXT J
1380 IGA=INT(DKAIMAX)
1390 IF A$="N" GOTO 1420
1400 FOR J=0 TO IGA:LINE(630-CINT(MSA*J),345)-(630-CINT(MSA*J),348):NEXT J
1410 GOTO 1430
1420 FOR J=0 TO IGA:LINE(40+CINT(MSA*J),345)-(40+CINT(MSA*J),348):NEXT J
1430 LOCATE 0,25,0
1440 IF A$="N" GOTO 1470
1450 FOR J=IGA TO 0 STEP -1:PRINT TAB(77-INT(J*MSA/8));:PRINT -J;:NEXT J
1460 GOTO 1480
1470 FOR J=0 TO IGA:PRINT TAB(4+INT(J*MSA/8));:PRINT J;:NEXT J
1480 FOR J=0 TO IGO
1490 LOCATE 0,2+INT(MSO*J*DGR/16),0:PRINT MAXGR-J*DGR;
1500 NEXT J
1510 LOCATE 0,1,0:PRINT "ΔlogX";
1520 IF A$="N" THEN LOCATE 18,24,0:PRINT "Δχ" ELSE LOCATE 18,24,0:PRINT ""
1530 FOR J=0 TO I
1540 IF IW(J)=0 GOTO 1590
1550 IF A$="N" GOTO 1570
1560 IPX=630+CINT(MSA*DKAI(J)):IPY=25+CINT(MSO*(MAXGR-DX(J))):GOTO 1580
1570 IPX=40+CINT(MSA*DKAI(J)):IPY=25+CINT(MSO*(MAXGR-DX(J)))
1580 CIRCLE(IPX,IPY),3:PAINT(IPX,IPY)
1590 NEXT J
1600 IF A$="I" GOTO 1670
1610 IPX1=630:IPY1=25+CINT(MSO*(MAXGR-AA-THETA*DKAIMAX))
1620 IF AA>MAXGR+.5/MSO THEN IPY2=25:IPX2=40+CINT(MSA*(MAXGR-AA)/THETA):GOTO 1650
1630 IF AA<MINGR-.5/MSO THEN IPY2=345:IPX2=40+CINT(MSA*(MINGR-AA)/THETA):GOTO 1650
1640 IPX2=40:IPY2=25+CINT(MSO*(MAXGR-AA))
1650 LINE(IPX1,IPY1)-(IPX2,IPY2)
1660 RETURN
1670 IPX1=40:IPY1=25+CINT(MSO*(MAXGR-AA+THETA*DKAIMAX))
1680 IF AA>MAXGR+5/MSO THEN IPY2=25:IPX2=630-CINT(MSA*(AA-MAXGR)/THETA):GOTO 1710
1690 IF AA<MINGR-5/MSO THEN IPY2=345:IPX2=630-CINT(MSA*(AA-MINGR)/THETA):GOTO 1710
1700 IPX2=630:IPY2=25+CINT(MSO*(MAXGR-AA))
1710 LINE(IPX1,IPY1)-(IPX2,IPY2)
1720 RETURN
1730 *GRAPH2
1740 CLS 3:SCREEN 2,0:COLOR 0:CONSOLE 0,25,0,0
1750 FOR J=0 TO I:LGXA(J)=LGX(J)+THETA*DKAI(J)+AA+V+.052:NEXT J
1760 LINE(20,360)-(630,360)
1770 LINE(20,20)-(20,360)
1780 FOR J=0 TO 3:LINE(17,20+J*100)-(20,20+J*100):NEXT J
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1790 FOR J=0 TO 6:LINE(30+J*100,360)-(30+J*100,363):NEXT J
1800 LOCATE 3,22,0:PRINT TAB(69);:PRINT "LogC"
1810 LOCATE 2,25,0:FOR J=0 TO 6:PRINT TAB(2+INT(J*12.5));:PRINT J-1;:NEXT J
1820 LOCATE 3,0,0:PRINT "Log(W/(2RΔλ))";SPC(18);"log2α = ";AALFA;SPC(8);"log(c/2RV) = ";V
1830 FOR J=0 TO 3:LOCATE 0,1+INT(6.25*J),0:PRINT 2-J:NEXT J
1840 PPY=2
1850 YY=PPY:GOSUB *KEISAN
1860 IF X>5 THEN PPY=PPY-.1:GOTO 1850
1870 IPX1=CINT(100*X)+130:IPY1=220-CINT(100*PPY)
1880 PPY=PPY-.1:YY=PPY:GOSUB *KEISAN
1890 IF PPY<-1.4 OR X<-1.21 GOTO 1930
1900 IPX2=CINT(100*X)+130:IPY2=220-CINT(100*PPY)
1910 LINE(IPX1,IPY1)-(IPX2,IPY2)
1920 IPX1=IPX2:IPY1=IPY2:GOTO 1880
1930 FOR J=0 TO I
1940 IPX=CINT(100*LGXA(J)):IPY=CINT(100*LGW(J))
1950 IF IPX>500 OR IPY>200 GOTO 1990
1960 IF IPX<-110 OR IPY<-140 GOTO 1990
1970 IPX=IPX+130:IPY=220-IPY
1980 CIRCLE(IPX,IPY),3:PAINT(IPX,IPY)
1990 NEXT J
2000 RETURN
2010 *KEISAN1
2020 REM Gradient of the curve is calculated from log(W/(2・R・Δλ))
2030 IF YY<TATE(0) THEN DC=9/(TATE(0)+10.052):RETURN
2040 FOR K=0 TO 11
2050 IF YY<=Y(K) GOTO 2080
2060 NEXT K
2070 DC=2:RETURN
2080 DC=2*A(K,0)*YY+A(K,1):RETURN
```