Reply to "Comment on 'Origin of light-induced states in intense laser fields and their observability in photoelectron spectra' "

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The preceding Comment has clarified a conjecture made in our paper. In this Reply, we add further insight to the results presented in the Comment concerning the behaviors of pole trajectories outside the resonance sectors.

Our calculation [1] was based on the complex-scaling method. By this method, one can obtain only the positions of poles in the resonance sector. In the weak field regime, the pole associated with a light-induced state (LIS) is located outside the resonance sector, and one cannot directly identify the origin of the LIS on the basis of the complex-scaling method. In Fig. 1 of Ref. [1], we extrapolated the pole trajectory by inspection. This conjecture gave a simple and unified explanation for the behaviors of the pole trajectories for different values of ω.

In the preceding Comment [2], Stroe and Boca directly calculated the pole positions outside the resonance sector, and have corrected our conjecture about the origin of the LIS in the case of $\omega = 0.55$. The LIS originates from the antibound state located on the channel threshold. We agree with their conclusion. In the cases of $\hbar\omega \lesssim 0.45$, the results of Stroe and Boca agree with our conjecture.

On the other hand, we would like to point out the peculiarity of the modified Pöschl-Teller potential, which possesses an antibound state at zero energy [2]. This situation is not generic but a peculiar case. No realistic atomic and molecular systems have an antibound state at zero energy.

In order to examine the generic behavior of LIS's, we have studied the model in which the energy positions of antibound states are controlled. It is found that the origin of the LIS is either an antibound state pole or a shadow pole depending on the shape of the potential.

We have investigated the square well potential with depth V and width $2a$. When V and a satisfy

$$\tan(a\sqrt{2V-1}) = \frac{1}{\sqrt{2V-1}}, \qquad (1)$$

the potential has a bound state at $E_0 = -1/2$. The energies of the antibound states depend on the potential strength $\gamma = a\sqrt{2V}$. Under the condition $\gamma < \pi/2$, there is only one antibound state. When one varies V and a so as to keep Eq. (1), the energy of the antibound state E_a changes, while E_0 remains unchanged.

The Floquet S matrix was analytically obtained by connecting local eigenfunctions (for example, [3]) inside and outside of the square well. Energy is analytically continued to the complex plane, and the poles of the S matrix elements are searched numerically over all the Riemann sheets. By varying the ponderomotive radius α, we traced the pole positions and obtained the pole trajectories. The laser frequency is set to $\hbar\omega = 0.55$.

We first examine the case of $V = 1.1019$ and $a = 0.673\,61$. This choice leads to $\gamma = 1$, $E_0 = -0.5$, and $E_a = -1.1019$. The antibound state is located at the bottom of the potential well, i.e., the relation $-V = E_a < E_0$ holds. As shown in Fig. 1, we identified three pole trajectories $\Phi_0^{(---)}$, $\Phi_0^{(++-) \to (--+)}$, and $\bar{\Phi}_{-1}^{(++-)}$; $\Phi_0^{(---)}$ originates from the original bound state (OBS), $\Phi_0^{(++-) \to (--+)}$ from the shadow pole of the OBS, and $\bar{\Phi}_{-1}^{(++-)}$ from the antibound state. The resonance state $\Phi_0^{(---)}$ originating from the OBS goes outside the resonance sector at $\alpha = 0.4$ but comes back at $\alpha = 0.6$. On the way, $\Phi_0^{(---)}$ exhibits a kind of avoided crossing with $\Phi_0^{(++-) \to (--+)}$ originating from the shadow pole. The pole $\Phi_0^{(++-) \to (--+)}$ is initially located on the Riemann sheet $(++-)$. After the avoided crossing with $\Phi_0^{(---)}$, $\Phi_0^{(++-) \to (--+)}$ enters the resonance sector on the sheet $(--+)$ at $\alpha = 0.45$. This means that $\Phi_0^{(++-) \to (--+)}$ becomes the LIS pole. On the other hand, the pole $\bar{\Phi}_{-1}^{(++-)}$ originating from the antibound state does not play any physical role. In this case, the origin of the LIS is a shadow pole as we discussed in Ref. [1].

Next we show the case in which the origin of the LIS is the antibound state. The potential parameters are chosen as $V = 0.827\,96$ and $a = 1.099\,00$. This leads to $\gamma = \sqrt{2}$, $E_0 = -0.5$, and $E_a = -0.0263$. In this case, $E_a > E_0$ holds in contrast to the preceding example. The pole trajectories are shown in Fig. 2. The pole $\bar{\Phi}_{-1}^{(++-) \to (--+)}$ enters the resonance sector, and gives rise to a LIS; namely, the origin of the LIS is the antibound state. This situation is similar to the case of $\hbar\omega = 0.55$ discussed in the Comment of Stroe and Boca.

The above observation implies that the origin of the LIS is the shadow pole when the antibound state is located deeply in the well. This situation is realized when the potential strength γ is smaller than a critical value γ_c. When $\gamma > \gamma_c$, the origin of the LIS is switched to the antibound state. The critical value of γ_c is found to be in the region $\gamma = 1.1153 - 1.1231$. At $\gamma = \gamma_c$, it holds that $E_a = E_0 + \Delta$, where $\Delta = 0.0167 - 0.0439$. In short, the origin of the LIS is the shadow pole when $E_a < E_0 + \Delta$.

We can restate a conjecture: the LIS originates from a shadow pole of the OBS when (i) $\hbar\omega < |E_0|$; or (ii) $\hbar\omega$ is in the vicinity above $|E_0|$, and at least $E_a \lesssim E_0$. In such cases, it is easier to detect the LIS in the photoelectron spectrum than

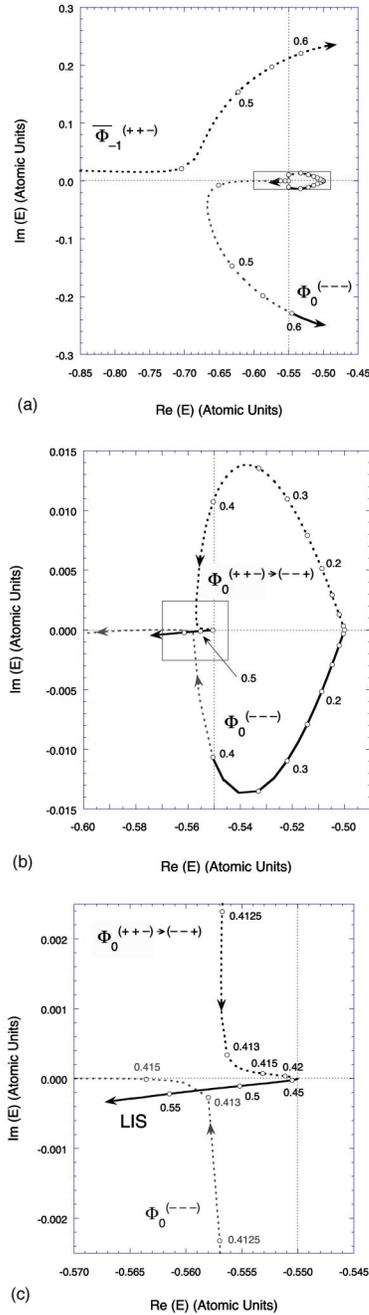*Author to whom correspondence should be addressed.

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(a)

(b)

(c)

FIG. 1. (a) Pole trajectories on the complex-energy plane with changing α. The values of the parameters are $\omega=0.55$, $V=1.1019$, and $a=0.673\,61$. The starting point of the pole trajectory $\bar{\Phi}_{-1}^{(++-)}$ is -1.6519. (b) Magnification of the part of (a) indicated by a rectangle (c) Magnification of the part (b) indicated by a rectangle. Two pole trajectories $\Phi_0^{(---)}$ and $\Phi_0^{(++-)\rightarrow(-+)}$ undergo a kind of avoided crossing with each other at $\alpha=0.413$. The pole trajectory $\Phi_0^{(++-)\rightarrow(-+)}$ makes a quick turn around the branch point $E=-0.55$.
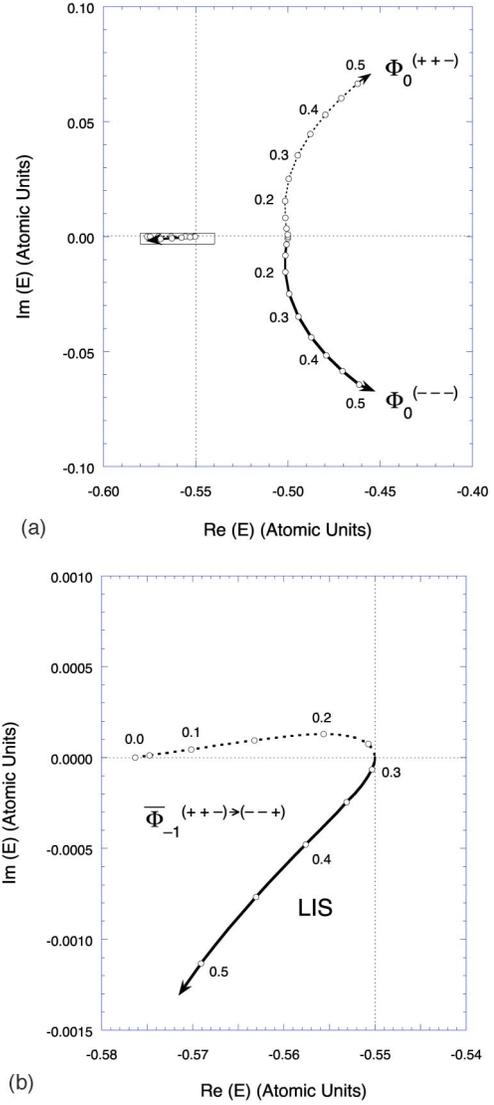

(a)

(b)

FIG. 2. (a) Pole trajectories on the complex-energy plane with changing α. The values of the parameters are $\omega=0.55$, $V=0.827\,96$, and $a=1.099\,00$. The starting point of the pole trajectory $\bar{\Phi}_{-1}^{(++-)}$ is -0.5763. (b) Magnification of the part of (a) indicated by a rectangle. The trajectory makes a quick turn around the branch point $E=-0.55$.

in the case of $\hbar\omega\gg|E_0|$. Previous results of Wells *et al.* [4] are also in this regime. In the case of $\hbar\omega>|E_0|$ and $E_a>E_0$, the antibound state is likely to be the origin of the LIS.

Finally, we should append that we have used the appropriate scalar product [5],

$$\langle\langle f|g\rangle\rangle = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt' \int_{-\infty}^{+\infty} dp\, f(p,t)g(p,t) \qquad (2)$$

in the actual calculation.

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